ABSTRACTS

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METHODS OF NONLINEAR ANALYSIS IN DIFFERENTIAL AND INTEGRAL EQUATIONS

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METHODS OF NONLINEAR ANALYSIS IN DIFFERENTIAL AND INTEGRAL EQUATIONS

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Four Operators – Four Spaces

Jürgen Appell

Department of Mathematics
University of Würzburg
Emil-Fischer-Straße 30
D-97074 Würzburg
Germany

Let X be a space of functions $x : [0,1] \to \mathbb{R}$ (to be specified below). In this talk we are going to consider the following four **operators**: The (linear) *multiplication operator* $M_\mu : X \to X$ defined by $(M_\mu x)(t) := \mu(t)x(t)$, where $\mu : [0,1] \to \mathbb{R}$, the (linear) *substitution operator* $\Sigma_\varphi : X \to X$ defined by $(\Sigma_\varphi x)(t) := x(\varphi(t))$, where $\varphi : [0,1] \to [0,1]$, the (nonlinear) *composition operator* $C_f : X \to X$ defined by $(C_f x)(t) := f(x(t))$, where $f : \mathbb{R} \to \mathbb{R}$, and the (nonlinear) *superposition operator* $S_g : X \to X$ defined by $(S_g x)(t) := g(t, x(t))$, where $g : [0,1] \times \mathbb{R} \to \mathbb{R}$.

The four **spaces** we will work in are the Banach space $X = C[0,1]$, equipped with the norm $\|x\|_C := \max\{|x(t)| : 0 \le t \le 1\}$, the Banach space $X = C^1[0,1]$, equipped with the norm $\|x\|_{C^1} := |x(0)| + \|x'\|_C$, the Banach space $X = Lip[0,1]$, equipped with the norm $\|x\|_{Lip} := |x(0)| + lip(x;[0,1])$, and the Banach space $X = BV[0,1]$, equipped with the norm $\|x\|_{BV} := |x(0)| + var(x;[0,1])$.

The **properties** of the four operators $A \in \{M_\mu, \Sigma_\varphi, C_f, S_g\}$ in the four spaces $X \in \{C, C^1, Lip, BV\}$ we are interested in are injectivity, surjectivity, boundedness, continuity, Lipschitz continuity, and compactness. Despite their simple form, these operators exhibit many surprises and pathologies. This is illustrated by a collection of 20 counterexamples which might be useful for your lectures and seminars on nonlinear analysis.

Wavefronts for diffusion-convection reaction equations with sign-changing diffusivity

Diego Berti

Department of Sciences and Methods for Engineering
University of Modena and Reggio Emilia
Via Amendola 2, Pad. Morselli
Reggio Emilia
Italy

We deal with *wavefront* solutions $\rho = \rho(t,x)$ to the following diffusion-convection reaction equation

$$\rho_t + f(\rho)_x = \{D(\rho)\rho_x\}_x + g(\rho), \qquad t \geq 0,\, x \in \mathbb{R}, \tag{1}$$

where the diffusivity D changes sign once, making (1) a *forward-backward* degenerate parabolic equation, and the reaction term g is of *monostable* type. Wavefronts for (1) are traveling wave solutions (so that ρ takes the form $\rho(t,x) = \varphi(x-ct)$, for some wave profile φ and wave speed $c \in \mathbb{R}$) whose profile is globally defined, non-constant and monotone.

The main result in [1], presented here, is that Equation (1) admits wavefronts (with decreasing profile) connecting the equilibria of g, and which cross regions where D changes sign, if and only if c is greater than or equals a proper threshold speed. Our approach relies on pasting together suitable semi-wavefronts defined in regions where D has a constant sign.

The wavefronts constructed in this way are unique (up to spatial shifts). Moreover, results about the strict monotonicity of profiles and characterization of when they are *sharp* (not globally differentiable) are provided. In particular, where the diffusivity changes sign, the sharp behaviours are new and unusual. Also, rather precise bounds on the critical speeds are discussed.

All results are based on joint work with Andrea Corli (University of Ferrara) and Luisa Malaguti (University of Modena and Reggio Emilia).

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On variation functions and their moduli of continuity

Simon Breneis

Stochastic Algorithms and Nonparametric Statistics
Weierstraß Institute for Applied Analysis and Stochastics
Mohrenstraße 39
10117 Berlin
Germany

We study the moduli of continuity of functions of bounded variation and of their variation functions. It is easy to see that the modulus of continuity of a function of bounded variation is always smaller or equal to the modulus of continuity of its variation function. We show that we cannot make any reasonable conclusion on the modulus of continuity of the variation function if we only know the modulus of continuity of the parent function itself. In particular, given two moduli of continuity, the first being weaker than Lipschitz continuity, we show that there exists a function of bounded variation with minimal modulus of continuity less than the first modulus of continuity, but with a variation function with minimal modulus of continuity greater than the second modulus of continuity. In particular, this negatively resolves the open problem whether the variation function of an α-Hölder continuous function is α-Hölder continuous.

Existence and uniqueness of positive solutions to a class of singular integral boundary value problems of fractional order with parametric dependence

Josefa Caballero Mena

Department of Mathematics
University of Las Palmas de Gran Canaria
Campus de Tarifa Baja, CP 35017
Las Palmas de Gran Canaria
Spain

In this paper, we study the existence and uniqueness of positive solutions for the following singular boundary value problem

$$\begin{cases} D_{0^+}^{\alpha} u(t) + f(t, u(t), (Hu)(t)) = 0, \quad 0 < t < 1, \\ u(0) = u'(0) = \ldots = u^{(n-2)}(0) = 0, \\ u(1) = \lambda \displaystyle\int_0^1 u(s)ds, \end{cases} \tag{2}$$

where $n \geq 3$, $n - 1 < \alpha \leq n$, $\lambda \in (0, \alpha)$, H is an operator applying $C[0, 1]$ into itself and $f : (0, 1] \times [0, \infty) \times [0, \infty) \to [0, \infty)$ (f can have a singularity at $t_0 = 0$) and $D_{0^+}^{\alpha}$ denotes the Riemann-Liouville fractional derivative. The main tool used in the proof of the results is a recent fixed point theorem in complete metric spaces.

On Two Questions of Measure of Noncompactness

Lixin Cheng

Department of Mathematics
School of Mathematical Sciences
Xiamen University
Xiamen 361005
China

In this talk, we will give the following two problems related to measure of noncompactness (NMC) affirmative answers.

Problem 1. Do there exist inequivalent regular MNCs on every infinite dimensional Banach space?

Problem 2. Let α be Kuratowski's MNC defined on a metric space M. Is the following statement true?

Given a nonempty bounded set $B \subset M$, there exists a countable subset $B_0 \subset B$ such that

$$\alpha(B_0) = \alpha(B).$$

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Poincaré inequalities and Neumann problems for the degenerate and variable exponent p-Laplacian

David Cruz-Uribe

OFS
The University of Alabama

In this talk we explore the relationship between the Poincaré inequality and solutions to a Neumann problem for the p-Laplacian. In the classical case, we show that the existence of the (p,p)-Poincaré inequality

$$\left(\int_E |f-f_E|^p\,dx\right)^{1/p} \leq C(p,E)\left(\int_E |\nabla f|^p\,dx\right)^{1/p},$$

where $f \in C^1(\overline{E})$ and $f_E = \frac{1}{|E|}\int_E f\,dx$, is equivalent to the existence of weak solutions to the Neumann problem

$$(N_p) \qquad \begin{cases} \Delta_p u = |f|^{p-2}f & x \in E \\ \mathbf{n}^t Q\nabla u = 0 & \text{on } \partial E, \end{cases}$$

where $\Delta_p u = -\operatorname{div}(|\nabla u|^{p-2}\nabla u)$.

Our central result is to show that under very weak (and natural) hypotheses the existence of a degenerate (p,p)-Poincaré inequality

$$\left(\int_E |f-f_E|^p v\,dx\right)^{1/p} \leq C(p,E)\left(\int_E |\sqrt{Q}\nabla f|^p\,dx\right)^{1/p}$$

is equivalent to the existence of weak solutions to the corresponding Neumann problem for the degenerate p-Laplacian

$$\Delta_{p,Q} u = -\operatorname{div}(|\sqrt{Q}\nabla u|^{p-2}Q\nabla u).$$

We also extend our work to the variable exponent setting and show that the existence of the variable exponent Poincaré inequality

$$\|(f-f_E)v\|_{L^{p(\cdot)}(E)} \leq C\|\sqrt{Q}\nabla f\|_{L^{p(\cdot)}(E)}$$

is equivalent to the existence of weak solutions to the corresponding Neumann problem for the degenerate p-Laplacian

$$\Delta_{p(\cdot),Q} u = -\operatorname{div}(|\sqrt{Q}\nabla u|^{p(\cdot)-2}Q\nabla u).$$

This research was conducted jointly with Scott Rodney (Cape Breton University), Emily Rosta (CBU), and Michael Penrod (University of Alabama). It was originally motivated by the work of E. Sawyer and R. Wheeden on degenerate elliptic operators, who assumed as a hypothesis the existence of such a degenerate (p,p)-Poincaré inequality.

Significance of Petryshyn's Fixed Point Theorem for the Existence results of Functional-integral Equations in Banach space

Amar Deep[a,1] and Deepmala[a,2]

[a]Mathematics Discipline
PDPM Indian Institute of Information Technology, Design and Management
Jabalpur-482005 (MP)
India

In this paper, we investigate the existence of solutions for some generalized functional-integral equations by Petryshyn's fixed point theorem in Banach space. Our result covers several earlier results obtained by various authors under some weaker conditions. We also provide some examples of functional-integral equations to verify the application of our established results.

Stability and Approximation of Almost Automorphic Solution on Time Scales for Stochastic Nicholson's Blow flies Model

Soniya Dhama

Department of Mathematics
CSA Gov PG Lead College Sehore M.P.
India

In this work, the stability and approximation of solutions of Nicholson's Blow flies model for stochastic case on time scales is addressed. Using various tools of analysis, sufficient conditions for the existence of square mean almost automorphic solution are derived. The randomness and time scales make the model as a hybrid which is more realistic and useful. The analysis works for both discrete and continuous case as well as for several other cases such as quantum and Cantor set. We establish appropriate conditions and results to explore Ulam-Hyers-Rassias stability. Furthermore, the model with piecewise constant argument is observed. Then the approximate solution and a nicer bound of this model using discretization method is established. At long last, we give an example to demonstrate the established results. The presented work is new and compliment the existing literature.

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A Generalization of Carathéodory Theorem Using Best Proximity Theory

Abhik Digar

Department of Mathematics
IIT Ropar
Rupnagar, Punjab
India- 140 001

Picard's existence and uniqueness theorem guarantees that the initial value problem(IVP)

$$y'(t) = f(t, y(t)); \quad y(t_0) = y_1$$

defined on a suitable rectangle R has a unique solution for some t-interval if the right hand side function f is continuous (in both variables) and Lipschitz continuous in second variable. Peano's theorem provides the existence of a solution of the IVP in absence of Lipschitz continuity. More generally, Carathéodory extension theorem ([2]) guarantees the existence of a solution of the same if the function f is measurable in the first variable and continuous in the second, and bounded by an L^1 function. In ([1]), authors proved the existence of a best approximate solution for the system

$$y'(t) = f_1(t, y(t)), y(t_0) = y_1; \ z_0(t) = f_2(t, z(t)), z(t_0) = y_2,$$

when f_1, f_2 are continuous on R (with some additional conditions). In this talk we discuss the best approximate solutions of a system of IVPs with right hand side functions measurable. To find such a best approximate solution we introduce a new class of functions called relatively continuous. As a consequence of our main theorem we obtain a weak solution of an IVP with right hand side function measurable.

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MEASURES OF NONCOMPACTNESS IN THE SPACE OF REGULATED FUNCTIONS ON AN UNBOUNDED INTERVAL

Szymon Dudek (joint work with Leszek Olszowy)

Faculty of Mathematics and Applied Physics
Rzeszów University of Technology
al. Powstańców Warszawy 8
Rzeszów
Poland

We formulate necessary and sufficient conditions for relative compactness in the space $BG(\mathbb{R}_+, E)$ of regulated and bounded functions defined on $\mathbb{R}_+$ with values in the Banach space E. Moreover, we construct four new measures of noncompactness in the space $BG(\mathbb{R}_+, E)$. Next, we present their properties and we describe relations between these measures.

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Travelling wave solutions for nonlinear differential equations on lattices

Michal Fečkan

Department of Mathematical Analysis and Numerical Mathematics
Faculty of Mathematics Physics and Informatics
Comenius University in Bratislava
Mlynská dolina
842 48 Bratislava
Slovakia

This is a survey based on some of my results with several coauthors [2, 3, 3, 4, 5, 5, 7, 6, 9] on travelling wave solutions for differential equations on lattices. We present metamaterials formed by discrete arrays of nonlinear resonators, implicit nonlinear lattices with applications in metamaterials, nonlinear magnetic metamaterials with parity-time-symmetric potentials, discrete nonlinear Schrödinger equations with local and nonlocal interactions, forced Fermi-Pasta-Ulam-type lattices and fractional differential equations on lattices. The presented results are rather broad and various.

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Existence of solutions to differential inclusions with primal lower nice functions

N. Fetouci and M. F. Yarou

Department of Mathematics
University of Jijel
BP 98 Ouled Aissa 18000
Jijel
Algeria

We prove the existence of absolutely continuous solutions to the differential inclusion

$$\dot{x}(t) \in F(x(t)) + h(t, x(t)),$$

where F is an upper semi-continuous set-valued function with compact values such that $F(x(t)) \subset \partial f(x(t))$ on $[0, T]$, where f is a primal lower nice function, and h a single valued Carathéodory perturbation.

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Distortions of isomorphic embeddings between L_1-preduals. Part II.

Agnieszka Gergont (joint work with Łukasz Piasecki)

Institute of Mathematics
Maria Curie-Skłodowska University
Lublin
Poland

I will talk about the distortion $\|T\| \, \|T^{-1}\|$ of an isomorphic embedding T of the space c of convergent sequences into an infinite-dimensional L_1-predual X such that $(\text{ext}B_{X^*})' \subset rB_{X^*}$ for some $r \in (0,1)$, that is, the set of all weak* cluster points of the set of all extreme points of the closed unit ball of the dual space X^* is contained in the closed ball of radius $r \in (0,1)$. I will present the proof that $\|T\| \, \|T^{-1}\| \geq (3-r)/(1+r)$. Moreover, for every $r \in (0,1)$, I will provide an example of an L_1-predual X with $(\text{ext}B_{X^*})' \subset rB_{X^*}$ for which this lower bound is, in fact, the Banach-Mazur distance.

This is a part of my Ph.D. thesis that is in preparation under the guidance of professor Łukasz Piasecki.

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Existence results for impulsive fractional differential equations with p-Laplacian via variational methods

John R. Graef

Department of Mathematics
University of Tennessee at Chattanooga
Chattanooga, TN 37403
USA

In this paper several sufficient conditions for the existence of at least one classical solution to impulsive fractional differential equations with a p-Laplacian and Dirichlet boundary conditions are presented. The approach is based on variational methods. Some recent results are extended and improved. An example to demonstrate the applicability of the results is presented.

Qualitative analysis of a nonlinear implicit neutral differential equation of fractional order

H. H. G. Hashem

Faculty of Science
Alexandria University
Alexandria
Egypt

Department of mathematics
College of Science
Qassim University
P.O. Box 6644 Buraidah 51452
Saudi Arabia

In this paper, we discuss suffcient conditions for the existence of solutions for a class of Initial value problem for an neutral differential equation involving Caputo fractional derivatives. Also, we discuss some types of Ulam stability: Ulam-Hyers stability, generalized Ulam-Hyers stability, Ulam-Hyers-Rassias stability and generalized Ulam-Hyers-Rassias stability for this class of implicit fractional-order differential equation. Some applications and particular cases are presented. Finally, the existence of at least one mild solution for this class of implicit fractional-order differential equation on an infinite interval by applying Schauder fixed point theorem and the local attractivity of solutions are proved.

Keywords: Neutral differential equation; Caputo fractional derivative; Ulam stability; Ulam-Hyers stability; Mild solution; Attractivity of solutions.

Boundary Value Problems for Third Order Ordinary Differential Equations

Johnny Henderson

Department of Mathematics
Baylor University
Waco, Texas 76798-7328
USA

For the third order ordinary differential equation, $y''' = f(t, y, y', y'')$, we consider "uniqueness implies existence" and "uniqueness conditions" for solutions satisfying the 2-point boundary conditions, $y(t_1) = y_1$, $y'(t_1) = y_2$, $y'(t_2) = y_3$, as well as the 2-point boundary conditions, $y'(t_1) = y_1$, $y(t_2) = y_2$, $y'(t_2) = y_3$, where $t_1 < t_2$. Uniqueness of solutions of such boundary value problems are intimately related to solutions of the third order equation satisfying certain 3-point boundary conditions. These relationships will be discussed as well. Finally, in the case when f is Lipschitz, optimal interval lengths will be determined, in terms of the Lipschitz coefficients, on which solutions are unique (hence exist), for the 2-point boundary value problems and the 3-point boundary value problems.

The handicap of metric fixed point theory

Erdal Karapinar

Cankaya University
Turkey

The aim of this talk is to reveal the dilemmas of the fixed point theorems that were started to be constructed by Banach in 1922. It is possible to transform many of the real-world problems into fixed point theorems. This shows how the fixed point theory is useful and how a huge application potential it has. On the other hand, a lot of results are repeated in this theory, which is of great interest. A substantial part of the new results suggested overlaps with the old results. To support this observation, I shall share a few examples from the current literature.

Impulsive Ψ–Hilfer Fractional Differential Equations

Kishor D. Kucche

Department of Mathematics
Shivaji University
Kolhapur-416 004, Maharashtra
India

In this talk, we discuss the formula for the solution of impulsive Ψ–Hilfer fractional differential equations. The existence and uniqueness of solution, Ulam–Hyers stabilities and data dependency results are discussed in the space of weighted piecewise continuous functions.

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Global Dynamics for an Age-Structured Model for HIV Viral Dynamics with Latently Infected T Cells

Manoj Kumar, Syed Abbas

School of Basic Sciences
Indian Institute of Technology Mandi
Kamand (H.P.) - 175005
India

This study focuses on an age-structured population model which describes the in host dynamics of HIV / AIDS. This model also considers the proportion of latently infected CD4+ T cells by HIV. After the formulation of the model, we derive steady-state solutions of the model. By constructing an appropriate Lyapunov function we check the stability of infection-free steady-state. We show that if reproduction number R_0 is less than 1, then the infection-free equilibrium point will be stable. We also study the role of anti-retroviral therapy to slow down the process of HIV in the human body. We consider our model with anti-retroviral therapy and again check the global stability of the infection-free steady state. We also solve our model numerically to study the primary spread of HIV virus.

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GROUP ANALYSIS OF A HAMILTON–JACOBI TYPE EQUATION

Jervin Zen Lobo

Department of Mathematics
St. Xavier's College
Mapusa – Goa
India

In this talk, we explain how Lie group analysis is applied to a Hamilton–Jacobi type equation. This equation under study is a first order nonlinear partial differential equation. We develop a Lie type invariance condition for first order nonlinear partial differential equations using local inverse theorem. This condition is then used to obtain the determining equations of the admitted Lie group. The determining equations are then split with respect to the arbitrary elements to obtain and solve the resulting splitting equations. We obtain the invariant surface condition, symmetry algebra and the group classification admitted by this equation. Further, we obtain the solutions of this equation and represent them graphically.

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A few properties of the superposition operator in the Banach space $CC(\mathbb{R}_+)$

Justyna Madej

Department of Nonlinear Analysis
Rzeszów University of Technology
Powstańców Warszawy 8
Rzeszów
Poland

We will indicate a few properties of the superposition operator considered in the Banach space $CC(\mathbb{R}_+)$ consisting of real functions defined, continuous on the real half-axis $\mathbb{R}_+$ and converging to finite limits at infinity. We will assume that the mentioned superposition operator F is generated by the function $f = f(t,x)$ satisfying three conditions:

i) The function $f(t,x)$ is continuous on the set $\mathbb{R}_+ \times \mathbb{R}$.

ii) The function $x \to f(t,x)$ is locally uniformly continuous with respect to the variable x uniformly for $t \in \mathbb{R}_+$.

iii) The function $t \to f(t,x)$ satisfies the Cauchy condition at infinity uniformly with respect to the variable x.

We will demonstrate our considerations with some examples.

Some Measures of Noncompactness and Their Applications

Eberhard Malkowsky

Faculty of Management
University Union Nikola Tesla
Njegoševa 1a
21205 Sremski Karlovci
Serbia

Measures of noncompactness are very useful tools in functional analysis, for instance in metric fixed point theory and the theory of operator equations in Banach spaces. They are also used in the studies of functional equations, ordinary and partial differential equations, fractional partial differential equations, integral and integro–differential equations, optimal control theory, and in the characterizations of compact operators between Banach spaces. We present an axiomatic introduction to measures of noncompactness on bounded subsets of complete metric spaces, and also the alternative axiomatic approaches by Banaś and Goebel and by Akhmerov et al. for measures of noncompactness in Banach spaces. As examples, we consider the Kuratowski, Hausdorff and separation measures of noncompactness and their most important properties. The Kuratowski measure of noncompactness is used in Darbo's fixed point theorem. Furthermore we study the notion of measures of noncompactness of operators between Banach spaces and some of their properties. Finally we give a few applications to the characterization of compact linear operators between certain BK spaces and to some results concerning the solvability of intergral equations.

Applications of differential transformation on some nonlinear partial differential equations

Anil Kumar and Giriraj Methi

Department of Mathematics & Statistics
Manipal University Jaipur
jaipur, Rajasthan, India

In this paper, differential transform is investigated on some higher-order nonlinear partial differential equations with initial and boundary conditions. Three problems are discussed to prove the effectiveness and reliability of the method. The comparison of numerical solution by Homotopy perturbation method (HPM) and exact solution are discussed. Error analysis is presented in detail. The present method is straight forward and rules out complex calculations inherent in other existing methods.

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Convergence methods and associated applications

S. A. Mohiuddine

Department of General Required Courses, Mathematics
Faculty of Applied Studies
King Abdulaziz University
Jeddah 21589
Saudi Arabia

Operator Theory and Applications Research Group
Department of Mathematics
King Abdulaziz University
Jeddah 21589
Saudi Arabia

We discuss the notions of statistical convergence, weighted statistical convergence, weighted equi-statistical convergence as well as other related notions and then present the relationship between these convergence methods. We also establish the Korovkin-type approximation theorems with the help of aforesaid convergence methods and construct some illustrative examples by taking positive linear operators which prove that our approximation theorems are stronger than some existing approximation results in the literature. Moreover, we discuss a Voronovskaya-type approximation theorem with the help of generalized weighted statistical convergence.

Nonuniqueness and blow up phenomena in the Volterra integral equations with the nondecreasing nonlinearity

Wojciech Mydlarczyk

Politechnika Wrocławska

We consider some classes of Volterra integral equations of the following form

$$(1) \qquad u(x) = \int_a^x k(x-s)g(u(s))ds, \quad x > a,$$

where either $a = 0$ or $a = -\infty$, $k(x)$ is a nonegative function and $g : [0, \infty) \to [0, \infty)$ is nondecreasing, continuous with $g(0) = 0$. The equation (1) has always a trivial solution $u \equiv 0$, but in some cases it has also nontrivial solutions $u \not\equiv 0$. We present necessary and sufficients conditions under which the equation (1) has nontrivial solutions $u(x)$. We also investigate the problem of the existence of the blowing up solutions where a solution $u(x)$ is called blowing up if $u(x) \to \infty$ as $x \to A$, for some $A < \infty$.

Functions connected with convexity and smoothness of normed spaces

Justyna Ochab

Rzeszów University of Technology
Powstańców Warszawy 8
Rzeszów
Poland

In this report, we will introduce several functions being moduli of convexity or smoothness or playing an intermediate role using moduli introduced by Gao and Lau ([7], [8]). We will indicate some similarities and relations between these functions and classical moduli of convexity and smoothness. Moreover, we will give the exact formulas of the mentioned functions in classical Banach spaces: $l^p, L^p(a,b), C([a,b])$, L^∞, l^∞ and we will introduce proofs for some of them.

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Existence results for singular elliptic equations with unbalanced growth

Florin-Iulian Onete

University of Craiova
Bucharest
Romania

We establish existence and multiplicity properties of solutions for a class of Dirichlet elliptic equations with double phase. The reaction contains the combined effects of a singular term and of a resonant perturbation. The proof combines variational tools from critical point theory and truncation and comparison techniques.

Some Novel Approaches of Fixed Point Theory in Modular Spaces

Mahpeyker ÖZTÜRK

Department of Mathematics
Sakarya University
Sakarya
Turkey

This study aims to reveal some new results about fixed point theory within the framework of spaces obtained by using modular and abstract metric functions. The study findings have the characteristics of generalizing, expanding and unifying various comparable results in the existing literature.

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Existence and uniqueness theorems for mixed initial and boundary value problems related to ordinary differential equations

Zsolt Páles

Institute of Mathematics
University of Debrecen
Egyetem tér 1
Debrecen
Hungary

In the talk we consider ordinary differential equations of the form

$$y'(x) = f(x, y(x)) \qquad (x \in I)$$

with the mixed initial and boundary value condition

$$\varphi(y(\xi_1), \ldots, y(\xi_m)) = 0,$$

where I is an interval, $f : I \times \mathbb{R}^n \to \mathbb{R}^n$ and $\varphi : \mathbb{R}^{nm} \to \mathbb{R}^n$ are continuous functions and $\xi_1, \ldots, \xi_m \in I$.

In our approach, we establish an equivalent integral equation and then apply the Banach Fixed Point Theorem combined with Bielecki's renorming method to obtain sufficient conditions for the unique solvability of the above problem.

Operator ideals on Banach spaces

Anna Pelczar-Barwacz

Institute of Mathematics
Jagiellonian University
Łojasiewicza 6
Kraków
Poland

A short overview of the recent results on the structure and cardinality of the lattice of closed ideals in the Banach algebra of bounded operators on a Banach space will be given. We will discuss in particular the case of certain sequence spaces, namely the Schlumprecht space and Schreier spaces, based on the joint work with A. Manoussakis.

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Distortions of isomorphic embeddings between L_1-preduals. Part I.

Łukasz Piasecki (joint work with Agnieszka Gergont)

Institute of Mathematics
Maria Curie-Skłodowska University
Lublin
Poland

I am going to introduce the topic and present known results devoted to the distortions of isomorphic embeddings between L_1-preduals. Then, I will announce the result obtained jointly with Agnieszka Gergont. Its proof will be given during Agnieszka Gergont's talk.

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Nonstandard phenomena in the study of anisotropic problems

Vicentiu Radulescu

Institute of Mathematics of the Romanian Academy
University of Craiova
Bucharest
Romania

We consider a class of non-standard PDEs with variable exponent and Dirichlet boundary condition. The feature of this analysis is that the considered problems has simultaneously a subcritical-critical-supercritical regime. We establish sufficient conditions for the existence of solutions and, in a singular setting, we prove a multiplicity property of solutions. The arguments combine the Palais principle of symmetric criticality with energy estimates, topological methods and critical point theory. Several open problems are raised in the final part of this talk.

Nonclassical Boundary Value Problems with BV-solutions

Simon Reinwand

Department of Mathematics
Julius-Maximilians-University Würzburg
Emil-Fischer-Straße 40
97074 Würzburg
Germany

We give criteria that guarantee the solvability of boundary value problems of the form

$$x''(t) = -\lambda g\big(t, x(t)\big) \quad \text{for } 0 \leq t \leq 1$$

subject to the nonclassical boundary values

$$x(0) = A_0 x \qquad \text{and} \qquad x(1) = A_1 x,$$

where A_0 and A_1 are bounded linear functionals on the space BV of functions $x : [0,1] \to \mathbb{R}$ of bounded (Jordan) variation.

This problem may be solved by transforming it into a Hammerstein integral equation. We therefore present first results solving the corresponding integral equation in the space BV based on Schauder's fixed point theorem and later show how to apply them to the original boundary value problem. The outcome will be a nice and simple formula that serves as a sufficient condition for the solvability and generalizes and simplifies known results. Some examples illustrate both the applicability and limits of our result.

SOLVABILITY OF A MIXED PROBLEM FOR A HEAT EQUATION WITH AN INVOLUTION PERTURBATION

Sarsenbi A.M.

Department of Informatics and Mathematics
International University SILKWAY
Tokayev street 27
Shymkent
Kazakhstan

We will consider the following mixed problem for a heat equation with involution perturbation of the form

$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2} - \alpha \frac{\partial^2 u(-x,t)}{\partial x^2} - q(x)\, u(x,t), \quad -1 < x < 1, \quad t > 0,$ (1)

$u(x,0) = \varphi(x), u'(-1,t) = 0, \quad u'(1,t) = 0,$ (2)

where $-1 < \alpha < 1$, the $q(x)$ is the real continuous function. We use the variable separation method : $u(x,t) = X(x)\, T(t)$. Here the function $X(x)$ gives the solution of the boundary value problem [1]

$-X''(x) + \alpha X''(-x) + q(x)\, X(x) = \lambda X(x), \ -1 < x < 1,$

$X'(-1) = X'(1) = 0.$

Theorem. Let the following conditions are satisfied:

1) the real function $q(x)$ is non-negative;

2) the number $\sqrt{\frac{1-\alpha}{1+\alpha}}$ is not even $(-1 < \alpha < 1)$.

3) the initial date $\varphi(x) \in C^2[-1,1]$ of the mixed problem (1), (2) satisfies the conditions

$\varphi'(-1) = \varphi'(1) = 0.$

Then the mixed problem (1), (2) has unique solution wich can be represented by the series

$u(x,t) = \sum\limits_{k=1}^{\infty} A_k e^{-\lambda_k t} X_k(x).$

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Existence and approximate controllability results for non-autonomous second order impulsive functional evolution equations with non local conditions

Soniya Singh[a,1] and Jaydev Dabas[a,2]

[a]Applied Science and Engineering
Indian Institute of Technology Roorkee
Roorkee, Uttarakhand 247667, India. Roorkee
India

In this paper, we are concerned with the approximate controllability results for second order non-autonomous impulsive functional evolution equations with nonlocal conditions in Banach spaces. First, we show the existence of a mild solution for the system under consideration with the help of the evolution family and application of Schauder fixed point theorem. Finally, we verify our results to examine the approximate controllability of the non-autonomous wave equation with non-instantaneous impulses and finite delay in the application section.

Existence and Multiplicity Results for Dirichlet Problem with Fractional Laplacian and Nonlinearity

Robert Stańczy

Instytut Matematyczny
Uniwersytet Wrocławski
Pl. Grunwaldzki 2/4
Wrocław, Polska

The existence and multiplicity results for the exterior Dirichlet BVPs for the equation involving the fractional Laplacian

$$(-\Delta)^{\alpha/2}u = \lambda f(u)$$

are established depending on the range of parameter λ and behavior of the nonlinearity f at zero and at infinity.

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On the conjecture of Schauder: a generalization and a new proof

Mohamed Aziz Taoudi

Cadi Ayyad University
Morocco

In this lecture, we prove that every nonempty compact s-convex subset ($0 < s \leq 1$) of a Hausdorff topological vector space has the fixed point property. Our approach leads to a simple alternative proof of Schauder's conjecture. This lecture is based on joint work with M. Ennassik.

Stability results for first-order nonlinear dynamic equations on time scales

Sanket Tikare

Department of Mathematics,
Ramniranjan Jhunjhunwala College,
Opposite Ghatkopar Railway Station, Ghatkopar (W),
Mumbai (M.S.)
India 400 086

We present the stability results for first-order nonlinear dynamic equations on time scales with local as well as nonlocal initial conditions. First, we obtain sufficient condition for Lyapunov stability. Then, we study various other stabilities using this Lyapunov stability theory. Some examples are given to illustrate the results obtained in this work.

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Uniform approximation of an impulsive differential system by using a piecewise constant argument on the halfline

Ricardo Torres Naranjo

Instituto de Ciencias Físicas y Matemáticas
Universidad Austral de Chile
Valdivia, Chile

Consider the differential equation with deviated argument

$$x'(t) = f\left([t/h]\,h, x\left([t/h]\,h\right)\right), \tag{3}$$

where $x_0 = x(kh)$, with $k \in \mathbb{Z}$, $h \in]0, \infty[$ fixed and $[\cdot]$ denotes the integer part function. We note that $[t/h]\,h = kh$ if $t \in I_k = [kh, (k+1)\,h)$. This function is a case of a piecewise constant function and it has jump discontinuities at the points $\mathbb{Z}h = \{kh : k \in \mathbb{Z}\}$. Then, if we set $t_k = kh$, integrating (3) in $I_k = [t_k, t_{k+1})$ and assuming continuity at $t = t_{k+1}$ we have $x\left(t_{k+1}\right) = x\big(t_k\big) + hf\big(t_k, x\left(t_k\right)\big)$. So, (3) can be seen as an approximating for $x'(t) = f(t, x(t))$, recovering the Euler's scheme.
Using the step function $\gamma(t) = [t/h]\,h$ and some stability hypothesis, we will approximate the solution of the following impulsive system

$$\begin{aligned} y_i'(t) &= -a_i(t)y_i(t) + \sum_{j=1}^{m} b_{ij}(t)f_j(y_j(t)) + c_i(t), \quad && t \neq t_k \\ \Delta y_i(t_k) &= -q_{i,k}y_i(t_k^-) + e_{i,k} + I_{i,k}(y(t_k^-)), && t = t_k, \end{aligned}$$

by the impulsive system with piecewise constant argument

$$\begin{aligned} z_i'(t) &= -a_i(t)z_i(t) + \sum_{j=1}^{m} b_{ij}(t)f_j(z_j(\gamma(t))) + c_i(t), \quad && t \neq \gamma(t_k) \\ \Delta z_i(\gamma(t_k)) &= -q_{i,k}z_i(\gamma(t_k)^-) + e_{i,k} + I_{i,k}(z(\gamma(t_k)^-)) && t = \gamma(t_k). \end{aligned}$$

The approximation will be uniform in the semiaxis $[\tau, \infty)$. I.e

$$\sup_{t \in [\tau, \infty)} |y(t) - z(t)| \to 0 \quad \text{as } h \to 0.$$

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Fixed points of monotone nonexpansive mappings

Andrzej Wiśnicki

Department of Mathematics
Pedagogical University of Krakow
30-084 Cracow
Poland

The recent results concerning fixed points of monotone nonexpansive mappings are discussed. Such results have found a lot of applications since the publication of Ran and Reurings in 2004.

In this talk the nonlinear ergodic theorem for monotone nonexpansive mappings in uniformly convex spaces is shown: if C is a bounded closed convex subset of an ordered uniformly convex space $(X, \|\cdot\|, \preceq)$, $T : C \to C$ a monotone 1-Lipschitz mapping and $x \preceq T(x)$, then the sequence of averages $\frac{1}{n}\sum_{i=0}^{n-1} T^i(x)$ converges weakly to a fixed point of T. As a consequence, the sequence of Picard's iteration $\{T^n(x)\}$ also converges weakly to a fixed point of T. The results are new even in a Hilbert space.

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Construction of some measures of noncompactness used in problem of solvability of an infinite system of nonlinear integral equations

Weronika Woś

The Faculty of Mechanics and Technology
Rzeszów University of Technology
ul. Kwiatkowskiego 4
37-450 Stalowa Wola
Poland

The subject of this work are infinite systems of nonlinear quadratic Volterra-Hammerstein integral equations. The main goal is to investigate the conditions sufficient for the existence of solutions of such systems, which are defined on the real half-axis $\mathbb{R}_+$. For this purpose there are applied Darbo's Fixed Point Theorem in conjunction with the theory of measure of noncompactness. Appropriate measures of noncompactness have been constructed in the $BC(\mathbb{R}_+, E)$ space consisting of defined, continuous and bounded functions on the real half-axis $\mathbb{R}_+$, with values in the given Banach space E. Moreover, we will assume that in the space E an axiomatically defined measure of noncompactness is given. In particular, we will consider the space of real bounded sequences l_∞ as the E space. The tools used in the work allow to obtain theorems about the existence of solutions to infinite systems of integral equations, which are defined on $\mathbb{R}_+$ and meet certain, imposed conditions.

The results are original and are based on the following publications:

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THE FORM OF LOCALLY DEFINED OPERATORS IN WATERMAN SPACES

Małgorzata Wróbel

Department of Mathematics
Czestochowa University of Technology
al. Armii Krajowej 21, PL 42-200
Częstochowa
Poland

An operator mapping certain class $\mathcal{G}$ of functions $f : [a, b] \to \mathbb{R}$ into another class $\mathcal{H}$ of functions defined on the same interval is said to be locally defined if two functions from the class $\mathcal{G}$ coincide on an open subset of $[a, b]$ then so do the images of the operator. In the talk I will give a representation formula for locally defined operators acting between Banach spaces of continuous functions of bounded variation in the Waterman sense. Moreover, the Nemytskij composition operators will be investigated and some consequences for locally bounded as well as uniformly bounded locally defined operators will be given.